Hub Network Design for Profit Optimization

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Presentation Outline

- Time definite LTL carrier’s operations
- Pure hub-and-spoke network
- The hub design directed network representation
- Hub network design problem
  - The mathematical model
  - The algorithmic design
    - Branch and bound
    - Implicit enumeration with embedded concave program
- Computational results
- Conclusions
LTL delivery industry

- *Time-definite* freight delivery common carriers
  - 3rd party logistics service providers
  - Publish tariff and serve general public
  - provide time-guaranteed door-to-door service for small shipment clients

- Operations network
  - *Local pickup/delivery*
    - To pickup freight from shippers and delivery to consignees
  - *Line-haul operations* in a hub-and-spoke network
    - To reduce partial loads
LTL operations networks

Line-haul operations

- feeders
- package cars
- shippers/congeinees

Local services

Local services
Hub-and-spoke networks

[Diagram showing hub-and-spoke network with centers 1, 2, 3, and 4 connected to a central hub.]
Pure hub-and-spoke networks

- **Facilities**
  - *Centers* are points of pickup and delivery
  - *Hubs* are points of consolidation

- **Tractor-trailer/Feeders**
  - Connect facilities by feeding freight
  - *Loads* with freight
  - *Empties* for equipment balancing

- **Pure hub-and-spoke networks**
  - All loads must either start or end at a hub
  - It reduces partial loads. With the slight increase in the handling costs, the overall cost is lower
Pure hub-and-spoke networks

- **Operational planning**
  - Commodity paths: freight in a sequence of facilities for an origin-destination (OD) pair
  - Vehicle (feeder) routes: tractor-trailer with various types on a sequence of links

- **Tactical planning**
  - Tariff chart/Prices for OD shipments

- **Strategic planning**
  - Hub network design
Hub network design w/cost min

- Time-definite LTL carriers
  - p-hub median problem
    - Campbell (2009); Lin et al., (2012)
  - Multimodal hub location problem
  - Stochastic p-hub center problem
    - Sim, et al., (2009)

- Time-indefinite LTL carriers
An example

- Inverse demand function $p=10-q$
- Revenue function $R=(10-q)q$
- Single vs Dual hubs
Cost minimization

Fixed demand (column 1): total cost for single hub, $10+$100; dual hub, 2*$10+$90.216

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<th>Single hub</th>
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<th>Dual hubs</th>
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Cost analysis:
- Single hub: Total cost = $10 + $100 = $110
- Dual hubs: Total cost = 2*$10 + $90.216 = $110.416
Profit Maximization

- If a fixed cost for a new hub is $10, resulted with $0.804 profit increased

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<td>price</td>
<td>u profit</td>
<td>Z profit</td>
<td>q</td>
<td>u cost</td>
<td>price</td>
<td>u profit</td>
<td>Z profit</td>
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The zone pricing

● Perception
  – The same time-in-transit shall be charged with the same tariff
  – Zip code – to – zip code; administrative division – to – administrative division

● Assumptions
  – The demand function is continuous, differential and downward slop
  – The revenue function is continuous, differential and concave
Zones

- Cluster centers in the same administrative division as Tier-1 zone
- Cluster adjacent zones into a hierarchical zoning system
Hub design w/ profit Optimization.

- Simultaneously determines
  - strategic hub locations
  - tactical zone prices (with associated demand)
  - an operation plan (paths)
  - to fill the available carrying capacity in a pure hub-and-spoke network while meeting the service levels and operational constraints

- To maximize its profit
Hub planning directed network

- A network representation on *activities*
- Nodes with times and costs
  - Each center has two nodes, *local sort* (for pickup) and *preload sort* (for delivery)
  - Each hub has as many *hub nodes* as consolidation
- Links
  - All the local sorts to hub sorts, all the hub sorts to hub sorts, and All the hub sorts to preload sorts
- Demands
  - A pair of *center local sort* to another *center preload sort*, an Origin-destination (OD) pair
Hub planning directed network

- **Center 1**: Local, Preload
- **Center 2**: Local, Preload
- **Center 3**: Preload, Local
- **Center 4**: Preload, Local

- **Hub-A**: Night
- **Hub-B**: Night

- **Tier-1 Zone**: Transportation Links
- **Tier-2 Zone**: Transportation Links
Hub design problem

Objective

\[ \max \Psi(q, x, z) = \sum_{od} \left[ \sum_m \delta_{od,m} (p_{m}^{m}(q_{m}^{m})q_{o}^{m}) - \left( \sum_{ij} (c_i + c_{ij})x_{ij}^{od} + c_d \right)q_{od}^{d} \right] - \sum_{h} C_{h}y_{h} \]

Flow conservation

\[ \sum_{j} x_{ji}^{od} - \sum_{j} x_{ij}^{od} = \begin{cases} 
-1, & \text{if } i = o; \\
1, & \text{if } i = d; \\
0, & \text{otherwise}; 
\end{cases} \]

Service level

\[ \sum_{ij} (t_i + t_{ij})x_{ij}^{od} + t_d \leq \hat{T}_{od} \]

Node capacity

\[ \sum_{odj} \left( \sum_{m} \delta_{od,m}q_{m}^{m} \right) x_{hj}^{od} \leq \hat{U}_{h}y_{h} \]

Directed in-tree

\[ \sum_{j} z_{ij}^{d} \leq 1 \]

Continuous/Binary

\[ q_{m}^{m} \in \mathbb{R}_{+}; y_{h}, z_{ij}^{d}, x_{ij}^{od} \in \{0,1\} \]
Directed in-tree

Hub-A

local
preload
center 1

night

preload
local
center 3

Hub-B

local
preload
center 2

night

transportation links

Hub-A

local
preload
center 1

night

preload
local
center 3

Hub-B

local
preload
center 2

night

transportation links
The branch-and-bound

- **Branch**
  - On the \( \{0,1\} \) hub location decision variables

- **Upper Bound**
  - Solve the zone pricing subproblem
  - With linear relaxation on *undetermined* hub location decision variable
  - Fathom whenever the upper bound is worse than the incumbent

- **Depth** search first
Zone pricing subproblem

**Objective**

\[
\max \Psi(q, x, z) = \sum_{od} \left[ \sum_{m} \delta^{od,m} \left( p^m(q^m)q^m \right) - \left( \sum_{ij} (c_i + c_{ij})x^{od}_{ij} + c_d \right)q^{od} \right] - \sum_{h \in H} C_h y_h
\]

**Flow conservation**

\[
\sum_j x^{od}_{ji} - \sum_j x^{od}_{ij} = \begin{cases} 
-1, & \text{if } i = o; \\
1, & \text{if } i = d; \\
0, & \text{otherwise}; 
\end{cases}
\]

**Service level**

\[
\sum_{ij} (t_i + t_{ij})x^{od}_{ij} + t_d \leq \hat{T}^{od}
\]

**Node capacity**

\[
\sum_{od} \sum_j \left( \sum_{m} \delta^{od,m} q^m \right) x^{od}_{hj} \leq \hat{U}_h
\]

\[
\sum_{od} \sum_j \left( \sum_{m} \delta^{od,m} q^m \right) x^{od}_{hj} \leq \hat{U}_h y_h
\]

**Directed in-tree**

\[
\sum_j z^{d}_{ij} \leq 1
\]

\[
\sum_{o} x^{od}_{ij} \leq Bz^{d}_{ij}
\]

**Continuous/Binary**

\[
q^m \in \mathbb{R}_+; z^{d}_{ij}, x^{od}_{ij} \in \{0,1\}
\]

\[
y_h, h \notin \overline{H}
\]
The largest LTL in Taiwan
Computational results

\[
\begin{align*}
Z_3 &= 1 & 73,910 \\
Z_3 &= 0 & 64,760 \\
Z_2 &= 1 & 77,382 \\
Z_2 &= 0 & 65,064 \\
Z_1 &= 1 & 78,542 \\
Z_1 &= 0 & 67,260 \\
\end{align*}
\]
## Complete enumeration

<table>
<thead>
<tr>
<th>Hub</th>
<th>Iteration</th>
<th>Revenue (NTD$)</th>
<th>Handling</th>
<th>Transport</th>
<th>Fixed Hub</th>
<th>Total (NTD$)</th>
<th>Profit (NTD$)</th>
<th>CPU (sec.)</th>
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<tbody>
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<td>2</td>
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<td>47,449.06</td>
<td>64,539.11</td>
<td>9,000.00</td>
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<td>58,967.53</td>
<td>0.618s</td>
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<td>223,970.50</td>
<td>68,152.13</td>
<td>73,058.37</td>
<td>18,000.00</td>
<td>159,210.50</td>
<td>64,760.00</td>
<td>55m40.487s</td>
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Conclusions

- Study the hub network design with profit optimization
  - **Reasoning**
  - Mathematical model, algorithmic design
  - With practical application on largest time-definite LTL carrier in Taiwan

- A higher number of hubs may NOT necessarily be negative to profit optimization

- Is it possible to apply to airlines/telecommunications industries?